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Catastrophic decays of compactified space-times

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Abstract: Witten long ago pointed out that the simplest Kaluza-Klein theory, without supersymmetry, is subject to a catastrophic instability. There are a variety of string theories which are potentially subject to these instabilities. Here we explore a number of questions: how generic are these instabilities? what happens when a potential is generated on the moduli space? in the presence of supersymmetry breaking, is there still a distinction between supersymmetric and non-supersymmetric states?

Keywords: Superstring Vacua, Classical Theories of Gravity.

Contents

1. Introduction

Witten long ago pointed out that the simplest Kaluza-Klein theory, without supersymmetry, is subject to a catastrophic instability. There are a variety of string theories which are potentially subject to these instabilities. Here we explore a number of questions: how generic are these instabilities? what happens when a potential is generated on the moduli space? in the presence of supersymmetry breaking, is there still a distinction between supersymmetric and non-supersymmetric states?

In studies of string theory in four dimensions, we often focus on string vacua which, at the classical level, have $N = 1$ supersymmetry. But it has never been clear why these should be particularly important. It is true that $N = 0$ vacua have potentials on moduli space at one loop, but the supersymmetric vacua have potentials non-perturbatively. The non-supersymmetric vacua often have tachyons in their moduli spaces, but it seems plausible that these theories might have AdS minima, somewhere on the interior of moduli space, and perhaps local dS or Poincare invariant minima elsewhere. In fact, even distinguishing these vacua seems to require some awkward phraseology, such as theories with "approximate moduli spaces with approximate $N = 1$ supersymmetry."

But various authors [1, 2, 3, 4] have noted one possible, sharp distinction: non-supersymmetric vacua sometimes suffer from a bizarre and catastrophic non-perturbative instability, known as Witten's "bubble of nothing" (BON) [1]. In these vacua, a bubble forms, just as in conventional false vacuum decay. This bubble grows at the speed of light, but the bubble wall, from the perspective of a four dimensional observer, is the end of space-time. The authors of refs. [2, 3] even speculated that this might be generic to non-supersymmetric states. Such bubbles do not seem to arise for supersymmetric solutions. It is natural to speculate that this represents a real distinction between supersymmetric and non-supersymmetric states. One might go so far as to suggest that the non-supersymmetric states do not make sense. Indeed, in the particular case of toroidal compactification, this was the viewpoint espoused by Brill and Horowitz [4] when they extended Witten's analysis to construct solutions with arbitrarily negative energy. If the existence of such bounce solutions (and their associated negative energy configurations) is more general, it is even tempting to argue that this might demonstrate that string theory *predicts* what we usually call low energy supersymmetry.

It is the purpose of this note to pursue these observations and speculations further. There are a number of objections which can (and have) been raised to these sorts of ideas. All of these must be addressed if one is to argue that the Witten bubble is a significant pathology which indicates that broad classes of string vacua are inconsistent — and perhaps to make a prediction of low energy supersymmetry:

1. Is the Witten solution, discovered by examining the lowest order classical equations of Einstein's gravity, a solution of string theory?

2. Is the Witten solution generic to non-supersymmetric solutions of string theory? At present, it can only be written down in a limited class of string/M theory models.

3. At the quantum level, the moduli are not exact, already at one loop. As a result, motion on the moduli space occurs on a much more rapid time scale than vacuum decay. So in what sense is it meaningful to discuss the Witten bounce in a quantum theory of gravity [6]?

4. If supersymmetry is spontaneously broken (by a small amount, as we imagine might be relevant to the description of the real world) does a Witten instability appear? In other words, is there any real distinction between classically supersymmetric vacua with four supersymmetries and non-supersymmetric vacua?

5. Finally, like other gravitational instantons, there is no sharp argument that the BON must be included in the semiclassical approximation to some underlying quantum theory of gravity. Perhaps it can simply be excluded from consideration, as can the negative energy configurations.

The goal of this work is to address these questions. In most cases, we will not be able to give definitive answers, but we will argue that it is quite plausible that the Witten solution

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is a solution of string theory, that it is exists in a broad class of non-supersymmetric strings, that it is relevant even in the presence of a potential for the moduli, and that it will not appear after (small) spontaneous supersymmetry breaking.

In the next section, we review the original Witten solution, and in section 3 we describe it in four dimensional terms, along the lines of the analysis of Coleman and DeLuccia (CDL) [5]. In section 4 we discuss the BON as a solution of string theory, arguing that, at least to all orders in the α' expansion, it is solves the classical equations, and that, in the non-supersymmetric case, there is a sensible, modular invariant conformal field theory which describes these configurations. We then consider the question of whether such instabilities are generic. Our starting point is the work of [2], in which non-supersymmetric configurations of the eleven-dimensional heterotic string were shown to admit BON solutions. We explain why it is difficult to prove that similar instabilities exist in general weakly coupled string theories — but why it seems likely that they do. We then turn to the question: what happens when moduli are stabilized. We start with the following observation: *if* some non-supersymmetric string theory solution describes nature, then necessarily there is a stable minimum for the moduli. We write down a model potential, which has stable moduli and the expected asymptotic behavior, and proceed to construct BON solutions, paying particular attention to the required boundary conditions. We find that such solutions generally exist, not only when the four dimensional space is flat, but also if it is de Sitter or Anti-de Sitter. We note that this semiclassical analysis is not likely to be reliable, but argue that it strongly indicates that the instability will be present in any such case. Finally, we explain why vacua with approximate supersymmetry (of the sort one expects if there is some sort of low energy supersymmetry) are not subject to these instabilities.

In the final section, we discuss possible ways to extend this analysis. Particularly interesting is the question of such bounce solutions in flux vacua [7, 8]. We also enumerate some puzzles raised by this work. In all of these cases, if we assert that the Witten solution does represent an instability, we have to ask how one might have gotten into the "false vacuum" in the first place. We argue that one possible way to understand this is in terms of the time-symmetric extensions of these solutions studied in [9]. Such solutions, however, are also subject to such instabilities, and their ultimate fate is not clear. Related issues include questions concerning the AdS-CFT correspondence. Finally, we speculate on the implications of the bubble of nothing for string phenomenology.

2. The Witten bounce

Coleman and Deluccia [5], many years ago, formulated the problem of vacuum decay in theories of gravity. They focused on theories in four dimensions, and considered decays of states with positive, zero and negative cosmological constants. Gravity, they discovered, introduces dramatic effects. For example, they found that often decay from flat space or de Sitter space to anti-de Sitter space does not occur, and when it does it leads to catastrophic gravitational collapse. Banks has recently revisited many of these issues, raising very general questions about vacuum decay in general relativity [6].

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Shortly afterwards, Witten wrote down a bounce solution of a different sort [1]. He noted that the euclidean Schwarschild solution of five dimensional gravity is a bounce, which describes the decay of the Kaluza-Klein vacuum, the compactification of five dimensional gravity on a circle of radius R. This solution is given by

$$ds^2 = \frac{dr^2}{(1 - \frac{R^2}{r^2})} + r^2 d\Omega_3^2 + \left(1 - \frac{R^2}{r^2}\right) dy^2\,. \tag{2.1}$$

Here y is the coordinate of the fifth dimension, $0 < y < 2\pi R$ and $d\Omega$ is the element of area of S^3. As we will discuss in greater detail shortly, from a four-dimensional perspective, the coefficient of dy^2 is a field, the radial modulus. This configuration has finite action,

$$S_o = \frac{\pi R^2}{4G}\,. \tag{2.2}$$

The singularity at $r = R$ is a coordinate singularity; in the neighborhood of $r = R$, the metric can be written in the form

$$ds^2 = du^2 + u^2 d\phi^2 + R^2 d\Omega_3^2\,, \tag{2.3}$$

with $y = R\phi$, i.e. the space looks like $R^2 \times S^3$.

Witten showed that this solution describes vacuum decay. To see the structure of the bubble which forms, one passes to Minkowski signature by taking polar angle of S^3, θ, and continuing:

$$\theta \to i\psi + \frac{\pi}{2}\,. \tag{2.4}$$

Then the metric becomes:

$$ds^2 = -d\psi^2 + \sinh^2(\psi) d\Omega^2 + \frac{dr^2}{(1 - \frac{R^2}{r^2})} + \left(1 - \frac{R^2}{r^2}\right) dy^2\,. \tag{2.5}$$

This describes a "bubble of nothing" (BON). It is a non-singular, geodesically complete space. $r = R$ is not a singular surface — it is, from the perspective of a four dimensional observer, the end of space-time.

Witten observed another striking feature of this solution: it is topology changing and it is not admissible in supersymmetric theories. To see this, note that if one does a 2π rotation near $r = R$, fermions pick up a minus sign. Since this region is smoothly connected to the large r region, the fermions in the original Kaluza-Klein space must obey anti-periodic boundary conditions in ϕ.

While a BON is a troubling end to a universe, it has never been clear whether one can simply discard theories with such instabilities. First, the actual fate of the universe is obscure. If such bubbles make sense, then they will be nucleated with a certain probability per unit volume per unit time, and the expanding bubbles will collide. Witten speculates that this situation will result in gravitational collapse (a two-bubble solution has been exhibited recently by Horowitz and Maeda [11]). Second, one might conjecture that, as we expect that string theory resolves at least some cosmological singularities, it might resolve these. If this is the case, then perhaps we live in such a universe, and its lifetime is just extremely long.

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However, Witten pointed out that the existence of these solutions is related to another seeming pathology of the Kaluza-Klein space: the energy is unbounded below. Explicit configurations of negative energy were exhibited in [4].

$$ds^2 = Ud\chi^2 + U^{-1}dr^2 + r^2 d\Omega^2 \tag{2.6}$$

where

$$U(4r) = 1 - \frac{2m}{r} - \frac{q^2}{r^2}\,. \tag{2.7}$$

The energy here is just m, which can have either sign. Not surprisingly, the zero time ($\tau = 0$) configuration in Witten's solution corresponds to the zero energy configuration (with $q = R$).

A second issue which has obscured the significance of the BON has to do with moduli. From the perspective a four dimensional observer, the bubble involves excitations of the graviton and a scalar, the radial modulus. In string theory, such a modulus will acquire a potential at one loop, and this will lead to motion on a scale rapid compared to the decay time. On the other hand, if such configurations have something to do with the world we see around us, there must be a stable minimum for the moduli potential. To analyze this problem, as well as to consider possible other solutions, it will be helpful to understand the Witten bubble from a four dimensional perspective.

3. The Witten bubble from a four dimensional perspective

One can try to formulate the Witten solution directly in the effective four dimensional field theory. Reducing from five dimensions to four, one has the four dimensional graviton, a scalar field, and a gauge field. In the solutions we will consider, the gauge field is not excited, and we will ignore it. We parameterize the five dimensional metric as

$$g_{55} = R^2(x) = Re^{\frac{2}{\sqrt{3}}\phi(x)} \tag{3.1}$$

(here ϕ is a four dimensional field, not to be confused with the five dimensional angle; similarly, $R(x)$ is a field; we will sometimes use the letter R to refer to this field and sometimes to the value of the radius in the would-be vacuum). The four dimensional effective action, after Weyl rescaling, has the form

$$S = \int d^4x\sqrt{g}\left(R + \frac{1}{2}\partial_\mu\phi\partial_\phi\phi g^{\mu\nu}\right). \tag{3.2}$$

In this form, we have a four-dimensional effective action of the type considered by CDL. Their action also includes a potential, $V(\phi)$. CDL look for a solution with $O(4)$ symmetry, which interpolates between the false and true vacua. The symmetry dictates that the metric may be written in the form:

$$ds^2 = d\xi^2 + \rho(\xi)d\Omega_3^2\,. \tag{3.3}$$

The euclidean equations for ϕ and ρ are:

$$\phi'' + \frac{3\rho'}{\rho}\phi' = \frac{dV}{d\phi}\,, \tag{3.4}$$

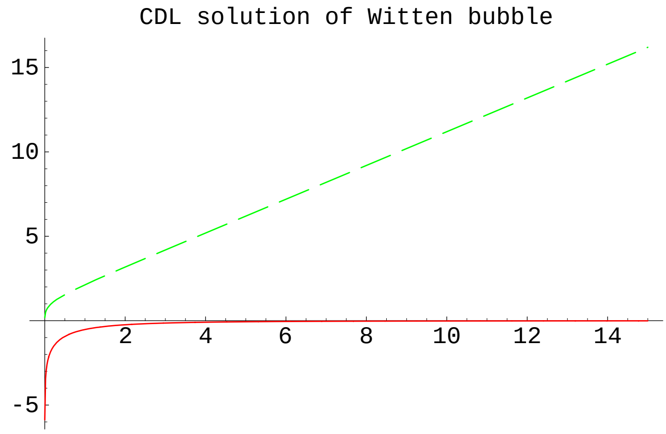

Figure 1: The Witten bubble as a CDL problem: the dotted line is $\rho(\xi)$ and the solid line is $\phi(\xi)$.

and

$$\rho'^2 = 1 + \frac{1}{3}\kappa\rho^2\left(\frac{1}{2}\phi'^2 - V\right). \tag{3.5}$$

CDL then consider an analog problem, where ξ plays the role of time. They require that at $\xi = 0$, the system starts off in (near) the true vacuum, while for $\xi \to \infty$ the system tends to the false vacuum. They also require that the system start off with zero velocity.

How does the Witten solution look in this description? For the Witten problem, the potential vanishes and the "true vacuum" lies infinitely far away (at $\phi = -\infty$). Weyl rescaling to four dimensions, the metric becomes:

$$ds^2 = \frac{dr^2}{\sqrt{1 - R^2/r^2}} + r^2\sqrt{1 - \frac{R^2}{r^2}}d\Omega^2\,. \tag{3.6}$$

To relate this to the CD equations, we need to solve

$$\frac{d\xi}{dr} = \left(1 - \frac{R^2}{r^2}\right)^{-1/4}. \tag{3.7}$$

Then we can read off ρ from the metric. In particular, as $r \to R$, we have

$$r = R + \left(\frac{3}{4}\left(\frac{2}{R}\right)^{1/4}\xi\right)^{4/3} \tag{3.8}$$

and

$$\rho = r\left(\frac{2(r-R)}{R}\right)^{1/4} \qquad \phi = \frac{1}{2}\sqrt{\frac{3}{2}}\ln 2\frac{r-R}{R}\,. \tag{3.9}$$

Note, in particular, that as $\xi \to 0$, the field and its derivative become large. In the analog mechanics problem, this corresponds to starting off with a big kick. It is a simple exercise with Mathematica to find the Witten solution from these equations, taking the limiting behavior above for the solution. Figure 1 shows the solution for ϕ and ρ as a function of

ξ, the CDL coordinate. The continuation to Minkowski signature is done just as in the five dimensional case. One takes the polar angle of S_3, and again makes the continuation $\theta \to i\frac{\pi}{2} + \psi$.

Written in this way, there is no particular obstacle to looking for BON solutions in the presence of a potential. It will be necessary, however, to pay close attention to the boundary conditions. We will do this shortly.

3.1 Geodesic motion

In order to understand the motion of the bubble wall, and light and matter in the presence of the bubble we need to know the geodesics for the spacetime. In the case of the Witten Bubble in flat space these have been calculated in [10]. As a warm up we rederive their result, using a method that is easily generalised to the AdS case. First, we assume, without loss of generality, that we fix one of the angles on the sphere, $\theta_1 = \pi/2$. We can then rewrite eq. (2.1) in a more convenient form using the following coordinate tranformations,

$$du = \sinh\psi d\theta - d\psi\,, \qquad dv = \sinh\psi d\theta + d\psi\,, \qquad z^2 = r^2 - R^2\,. \tag{3.10}$$

The metric is now,

$$ds^2 = (z^2 + R^2)dudv + dz^2 + \frac{R^2 z^2}{z^2 + R^2}dy^2 \tag{3.11}$$

There are a set of conserved momenta associated with geodesic motion,

$$(z^2 + R^2)\dot{u} = k_v\,, \qquad (z^2 + R^2)\dot{v} = k_u\,, \qquad \frac{R^2 z^2}{z^2 + R^2}\dot{y} = k_y \tag{3.12}$$

where dot corresponds to derivative with respect to the proper time along the geodesics. We normalize this affine parameter along the geodesics such that $\mu^2 \equiv \dot{x}_\alpha \dot{x}^\alpha$ is 0 (-1) for a lightlike (timelike) geodesic. Using the conserved quantities it is possible to write down a first order differential equation for the radial motion,

$$\dot{z}^2 + \frac{k_u k_v}{z^2 + R^2} + k_y^2 \frac{z^2 + R^2}{R^2 z^2} = \mu^2 = \begin{cases} 0: & \text{null} \\ -1: & \text{timelike} \end{cases} \tag{3.13}$$

This equation describes a particle moving in an effective potential,

$$V_{\rm eff} = \frac{k_u k_v}{z^2 + R^2} + k_y^2 \frac{z^2 + R^2}{R^2 z^2} - \mu^2 \tag{3.14}$$

For simplicity we will consider motion where the remaining angle, θ, is held fixed. In this case $k_u = -k_v$ and the first term in the effective potential is negative asymptoting to zero at large z, the second term is positive definite and tends to a constant, k_y^2/R^2, at large z. The existence of unbound motions then depends only on whether $k_y = 0$ or not. For $k_y = 0$ there exist unbound lightlike geodesics but regardless of the magnitude of k_y there are no unbound timelike geodesics.

A massive particle carries out oscillatory motion with turning points found by solving $V_{\rm eff} = 0$. In the case of no motion in the y direction, $k_y = 0$, the inner turning point[1] is

[1] Note that the transformation of eq. (3.10) is only valid for $r = R$ if $\dot{r} = 0$, so if the particle makes it into $z = 0$ it must stop there.

$z = 0$, the bubble wall. The bubble wall hits a massive particle which bounces off and moves ahead of the bubble. The wall accelerates and catches up, pushing the particle forward again. The maximum separation of the particle from the bubble wall is given by the larger root of $V_{\rm eff} = 0$. With $k_y \neq 0$ the particle has momentum in the compact extra dimension and the turning points are shifted. The inner turning point is shifted out from $z = 0$ and the outer is shifted in.

4. Bubbles of nothing in string/M-theory

4.1 The five dimensional solution as a solution of string theory

Witten's solution would seem to be a good solution of certain string theories. For example, Rohm long ago wrote down string solutions of the type II and heterotic theories with Scherk-Schwarz boundary conditions [12]. These have antiperiodic boundary conditions for the fermions in the fifth dimension, precisely what would seem to be required to support a Witten bubble. Higher order terms in the curvature will appear in the α' expansion, and will correct the solution. But there is no apparent obstacle to constructing a solution in powers of $1/R$ in the α' expansion. There are no dangerous zero modes. The solution is non-singular, as we saw above.

It is easy to check, in fact, that all curvature invariants are non-singular for the Witten solution. Certain components of the Riemann tensor for Witten's solution blow up or vanish as $r \to R$. For example:

$$R_{r\theta r\theta} = -\frac{R^2}{r^2 - R^2} \qquad R_{\theta\phi\theta\phi} = -\frac{(r^2 - R^2)R^2}{r^4} \tag{4.1}$$

So one might worry that terms in the action constructed out of R_{MNOP} might be singular. But, for instance, powers of $R^2_{r\theta r\theta}$ are accompanied by $(g^{rr})^2(g^{\theta\theta})^2$, and thus are not singular as $r \to R$. Instead the scalar quantitity built from $R_{r\theta r\theta}$ is $-1/R^4$ in the limit $r \to R$. Similarly, to build an invariant from the vanishing quantity $R_{\theta\phi\theta\phi}$ introduces powers, $(g^{\phi\phi})^2$, which is singular, but again the resultant invariant is $-1/R^4$. From the perspective of a four dimensional observer, this is rather miraculous. The four dimensional action involves combinations of the curvature and the moduli in just such a way that there are no singularities, no matter how many powers of fields and derivatives are included. From the perspective of the five dimensional observer, there is, of course, no miracle at all; as we saw, the singularity is a coordinate singularity.

We should also ask whether these are solutions non-perturbatively in α'. Here we would note, again, that there are not candidate zero modes which could serve as an obstruction to finding a solution. Still, it would be desirable to have another argument directly in conformal field theory.

Finally, we should worry about the dilaton and other moduli (besides the five dimensional radius). If we compactify, for example, a ten dimensional heterotic string on a six dimensional torus, where the torus has the structure $S_1 \times T_5$, the moduli space factorizes. The non-trivial field configuration for g_{55}, the radius of the circle, does not serve as a source for either the dilaton or the other moduli.

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4.2 Generalizations of the Witten solution

It is natural to wonder whether one can write such solutions for more general non-supersymmetric strings. In weakly coupled string theories, there are a large number of rather trivial generalizations, arising whenever one compact dimension is a circle with anti-periodic boundary conditions for fermions. The discussion of the previous section suggests another set of generalizations in weakly coupled string theory. We have written Witten's solution purely in terms of four dimensional fields. The action for the string dilaton is (apart from a numerical factor) identical to that for the field R. So we can simply transcribe our solution in terms of the four dimensional metric and R into a solution of the four dimensional metric and the dilaton. In this way, we can write down a host of possible solutions. For example, we might consider the $O(16) \times O(16)$ theory compactified on a Calabi-Yau space (such a configuration is a solution of the classical string equations; some of its features will be discussed elsewhere [13]). All of these would then admit a solution of the classical equations, at the level of the two derivative terms.

The difficulty with these solutions is that the four dimensional curvature is singular, and we have, a priori, no reason to expect a cancellation of all singular terms in the action by powers of the dilaton, as occurred in the five dimensional compactifications.

On the other hand, we know that at strong coupling, the dilaton is often the radius of a circle. Indeed, Fabinger and Hořava generated a large class of additional M-theory solutions, by considering the strongly coupled limit of the heterotic string [2]. In this case, the role of the "fifth" dimension is played by the circle on which the eleventh dimension is compactified. They noted that one can choose the chirality of the fermions on the two walls to be opposite, breaking all of the supersymmetry. One can then write down a Witten solution. Such a configuration is again non-singular. One can easily write down solutions when other dimensions are compactified. It is not clear, a priori, under what circumstances these configurations correspond to approximate solutions of some set of exact classical equations. For example, if one compactifies on a Calabi-Yau space, higher derivative operators will presumably destabilize the solution (even "classically"). But, in the spirit of our earlier discussion, the whole question of the bounce is not particularly interesting unless the moduli are somehow stabilized, in which case our considerations below will suggest that solutions exist. (Fabinger and Hořava speculated that solutions would exist if moduli were stabilized).

Returning to our weak coupling compactifications, one might conjecture that there is some sort of duality between the strongly coupled and weakly coupled non-supersymmetric strings, and that the BON solutions somehow make sense. Establishing such a duality is, of course, beyond present techniques. Indeed, if one could establish such dualities, one would have presumably established the non-perturbative consistency of these string states.

So in the weakly coupled limit,it is easy to establish that a broad class of flat, four dimensional configurations solve the string equations of motion, but it is difficult to establish whether a Witten instability exists. In the strongly coupled limit, it is more difficult to determine when configurations are solutions of the equations of motion, but it is eas-

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ier to argue that if they are, there is a non-singular Witten-type solution. But in light of these observations it would be surprising if these instabilities were not generic in non-supersymmetric string solutions.

5. Including a potential

In non-supersymmetric theories, a potential for the moduli appears already at one loop. This potential tends to zero as $R \to \infty$, the limit in which the theory becomes ten (or eleven) dimensional). It also tends to zero for small R. This behaviour of the potential for large and small R is computed only for weak string coupling. This is a reflection of the fact that in string/M theory, small radius is generally equivalent to large radius, possibly in some other string theory. If a non-supersymmetric solution of string theory is relevant to nature, the potential should have at least a stationary point. In this section we will consider model potentials, with a minimum of the energy at some value of R, and which tends to zero as $R \to \infty$ or $R \to 0$. An example of a potential is given in figure 2 (the figure is drawn so as to emphasize the stationary point, so the behavior at the origin is not visible). The Witten solution includes a region where the extra dimension shrinks to zero size, so the vanishing of the potential at small radius will be an important feature of the model.[2]

We again want to solve equations (3.4), (3.5). We need to think about the boundary conditions carefully. It is helpful to return, first, to five dimensions, and to study a metric of the form:

$$ds^2 = f(r)dr^2 + r^2 d\Omega_3^2 + h(r)d\phi^2 \,. \tag{5.1}$$

It is straightforward to compute the curvature for this metric. Vanishing of the Ricci tensor gives the equations:

$$\frac{f'}{f} = -\frac{h'}{h}f' - \frac{2}{r}f + \frac{2}{r}f^2 = 0 \,. \tag{5.2}$$

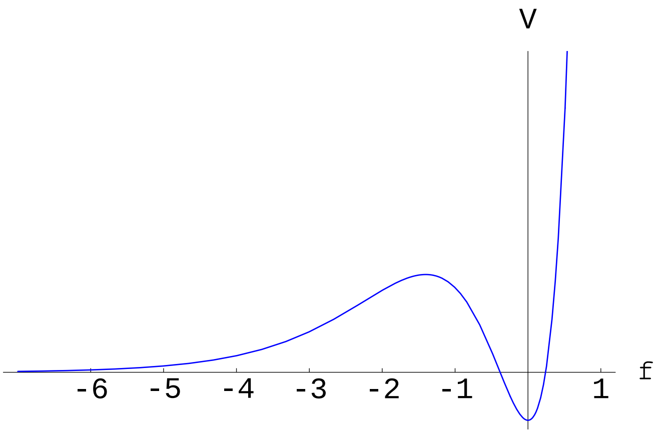

Figure 2: Example potential, $V(\phi)$.

[2]One can write down a T-dual of Witten's solution, in which the extra dimension blows up as $r \to R$. The existence of this solution raises some conceptual questions which will be dealt with elsewhere.

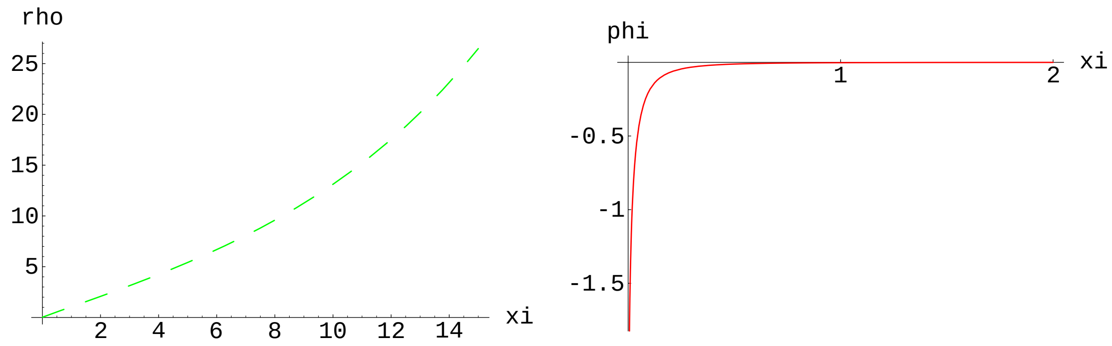

Figure 3: Numerical solution for the Witten bubble with a potential for the radion. The dotted line is $\rho(\xi)$ and the solid line is $\phi(\xi)$.

Assume that f has a singularity (a simple pole) at r_o,

$$f = a\frac{r_o}{2(r-r_o)} + \text{ constant}, h(r) = 2r_o(r-r_o)c. \tag{5.3}$$

Then near this point, we have:

$$-a + a^2 = 0 \qquad \text{so } a = 1\,. \tag{5.4}$$

$h(r)$ is determined by the requirement that the singularity is merely a coordinate singularity: This gives $c = 1$. Note that at time zero, this is a configuration of the type considered by Brill and Horowitz.

As boundary conditions at $\xi \to 0$, we require that ϕ and its first derivative behave as in the Witten solution. This insures that, even though the field and its derivative are blowing up, interpreted in five dimensions, the singularity is a coordinate singularity. We then ask whether we can find a solution which asymptotically tends to the stable vacuum (which we will loosely refer to as the "false vacuum"). We require that the metric behave as in eq. (5.2), and treat r_o as a parameter.

We study the bounce for a potential $V(\phi)$. We find a solution, following Coleman's dictum, by varying r_o so as to obtain both undershoot and overshoot of $\phi = 0$. We check that the metric is non-singular as $\rho \to \infty$. We find, for the potential of figure 2 there is a solution. An example is shown in figure 3. We find solutions when the "false vacuum" has positive, zero, or *negative* cosmological constant. This is, perhaps, not surprising, given the existence of configurations of arbitrarily negative energy in the flat Kaluza-Klein theory.

The existence of an AdS solution is particularly curious. Since light can reach the boundary of AdS in finite time, it is natural to ask whether the wall reaches the boundary in finite time. As we will explain shortly, the answer is yes. From the perspective of the AdS-CFT correspondence, the existence of such solutions is also puzzling. We will discuss these questions further in the concluding section.

5.1 Geodesics in AdS case

Following a similar analysis to the flat case we can reduce the discussion of geodesics in the AdS Witten bubble to an analysis of the classical motion of a particle in a potential. After solving the CDL problem in 4 dimensions one can lift the metric back up to 5 dimensions,

$$ds^2 = -\rho^2(\xi)dt^2 + d\xi^2 + \rho^2(\xi)\sinh(t)d\Omega_2^2 + R^2(\xi)dy^2 . \tag{5.5}$$

ρ and R are found numerically for a given potential for the radion, σ. Recall, for instance, that $R = e^{\sqrt{2/3}\sigma}$.

Using the first 2 transformations of eq. (3.10) we can get the metric into the same form as eq. (3.11), again making the assumption of fixing one of the angles on the sphere. We can identify conserved momenta and find the 'conserved energy' of the motion. We find,

$$\dot{\xi}^2 + \frac{k_u k_v}{\rho^2} + \frac{k_y^2}{R^2} = \mu^2 \tag{5.6}$$

with conserved momenta,

$$\rho^2\dot{u} = k_v , \qquad \rho^2\dot{v} = k_u , \qquad R^2\dot{y} = k_y . \tag{5.7}$$

Again one finds that a massive particle is struck by the wall. This occurs in a finite proper time, no matter how "close" the particle starts out to the boundary. The particle is then bound to the wall. It takes infinite proper time for the particle to reach the boundary, but, as we will explain in the next section, only a finite conformal time elapses until the wall reaches the boundary.

5.2 Action of the bubble

The tunnelling amplitude for the bubble decay is proportional to e^{-S} where S is the classical action of the bounce solution. For the Witten bubble this is finite and $S = \frac{\pi R^2}{4G_N}$. The contribution to this action from the usual bulk term is zero, because the solution is Ricci flat. However, a boundary term must be added to cancel second derivative terms this gives a non-zero contribution. In general the complete action is,

$$S = \frac{1}{32\pi^2 G_N R}\int d^5x\sqrt{g}R + \frac{1}{16\pi^2 G_N R}\int d^4x\sqrt{\gamma}K \tag{5.8}$$

γ is the metric on the boundary with unit normal $\hat{n}$ and $K = -\gamma^{ab}\nabla_a\hat{n}_b$ is its intrinsic curvature.

In going from the 5D setup to the 4D gravity-scalar system one must dimensionally reduce and Weyl rescale the metric. This results in additional contributions to both the boundary and bulk actions from the scalar. Thus,

$$\begin{aligned} S_4 = & \frac{1}{32\pi^2 G_N R}\int d^4x\sqrt{\tilde{g}}\left(\tilde{R} + \sqrt{\frac{2}{3}}\tilde{\nabla}^2\sigma - \tilde{g}^{ab}\partial_a\sigma\partial_b\sigma + V(\sigma)\right) + \\ & + \frac{1}{16\pi^2 G_N R}\int d^3x\sqrt{\tilde{\gamma}}\left(\tilde{K} + \frac{1}{2}\sqrt{\frac{2}{3}}\partial_r\sigma\sqrt{\tilde{g}^{rr}}\right) . \end{aligned} \tag{5.9}$$

One can check that, in the case where explicit results are known, this gives the same result for the Witten bubble as the 5D calculation. In the case of a non-zero potential, in flat space, the calculation must be done numerically. We have checked that this gives a finite result for the action. Likewise, the action for the AdS case must be calculated numerically but also produces a finite result.

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5.3 Spontaneously broken supersymmetry

Now consider supersymmetric string models. Here one expects that a potential appears non-perturbatively. By itself, this is not a qualitative difference from non-supersymmetric theories. But if supersymmetry is broken at low energies (as in gluino condensation and similar mechanisms), the effect is small, and is negligible for the non-zero Kaluza-Klein modes. The distinction between supersymmetric and non-supersymmetric solutions at the classical level has to do with properties of the non-zero modes, so under these conditions we might expect that, even when supersymmetry is broken, there will be no BON.

6. Implications and conjectures

One of the most urgent questions in string theory is whether or not low energy supersymmetry is a prediction. There are a number of obstructions to deciding this question. The first of these is distinguishing approximate supersymmetric states in string theory from those which are not supersymmetric in any approximation, or more precisely those which do not have only four supersymmetries in any limit of an (approximate) moduli space. The Witten bubble makes precisely such a distinction. Our studies, building on earlier observations of Fabinger and Hořava, *suggest*–and unfortunately at present the connection cannot be made stronger–that BON decays are truly solutions of string theory, and are generic to non-supersymmetric strings. We have also seen here that if we *assume* that moduli are stabilized, with vanishing cosmological constant, then typically decays occur. Vacua for which this is not the case are presumably not interesting for describing the real world. Finally, we have argued that even in the presence of small, spontaneous supersymmetry breaking, there is no BON instability in the supersymmetric case. If these conjectures are correct, then non-supersymmetric vacua with stable moduli and small cosmological constant would seem an unlikely outcome of string theory.

Our observations, however, suffer from serious limitations, and must, for the most part, be viewed as conjectural. Some can surely be sharpened, and this is the subject of continuing investigations. Carefully defining the appropriate conformal field theories appears possible. Some of the questions of decays in supersymmetric and non-supersymmetric states can be studied in flux compactifications, although we have not calculated how the Witten bubble solution is modified due to the flux backreaction.

However, whether these solutions are indicative of a real pathology or not is not clear. First, as always in considering quantum gravity, there is no sharp argument that it is necessary to include these topologically non-trivial configurations in some underlying path integral formulation of the theory. Perhaps they can simply be banished. We view the existence of a sensible euclidean conformal field theory as some evidence that these configurations are to be included, but this is hardly conclusive. Second, it could be that, as pathological as they appear, these pathologies are ultimately smoothed out in some underlying quantum theory. All that might be important for our universe is that our current state be very long lived.

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6.1 AdS puzzles

These puzzles and their possible resolutions are illustrated by the AdS solutions. These have a number of troubling features. First, the wall reaches the boundary in finite *conformal* time. To see this, consider a point on the leading edge of the wall. In the CDL coordinates, this can be a point of large, fixed ξ. For very large ξ, the CDL equations become the equations for AdS space in somewhat unusual coordinates. The transformation between the CDL and global AdS coordinates, for large ξ and ψ is:

$$\rho \approx \xi + \psi - \ln(2) + (e^{-2\psi} - e^{-2\xi}) \qquad \cos(\tau) \approx 2e^{-\psi}(1 - e^{-2\psi} + 2e^{-2\xi}) \,. \tag{6.1}$$

So we see explicitly that we reach $\rho = \infty$ at time $\tau = \pi/2$. Curiously, any massive observer is swept along by the wall, and only reaches the wall after an infinite *proper* time. Moreover, if this observer emits light rays, at large time they take a fixed (proper) time to reach the boundary. These observations suggest that it is not be appropriate to think of the bubble as a bubble in a true AdS space. One might, instead, think of starting with the time-symmetric solution (i.e. take the continuation of the euclidean solution, and allow $-\infty < \psi < \infty$), in which, in the far past, one starts, essentially, with nothing, and returns to that state in the far future [9].[3] In such a space-time, one can still form bubbles (this is true also for the Minkowski and dS cases), and these will grow, collide, etc. Whether there is a sensible quantum theory in such spaces is not clear. It seems quite possible that there is not.

There seem to us to be at least three possible resolutions.

- Stable AdS spaces are expected to have conformal duals. If such a dual for these candidate vacua exist, then there is a stable AdS state, and the solution we have found can not be interpreted as a gravitational instability of AdS. Indeed, since the bubble geometry is not asymptotically ADS, its quantum description–if one exists–is completely autonomous and we should not include the BON as a contribution to the gravitational path integral.

- This leaves the possibility described above, that the that the bubble of nothing is a finite-lived quantum system that contracts from nothing and then expands. Local geometry during the evolution outside of the bubble looks like ADS space. Perhaps there would be some relic of this state in the dynamics of the CFT.

- There might be no quantum completion of the classical solution. In this case the BON is neither an instability of ADS nor a sensible system in its own right; it is just an artifact of naive application of the semiclassical method.

- There is no CFT dual, and there are simply no sensible quantum systems in cases where a BON solution exists.

One can clearly add further speculations to this list. For the flat space and dS case, there are analogous possibilities.

[3]We thank Tom Banks for raising this possibility.

6.2 Future directions

There are other directions for further work, which we are pursuing. Flux vacua are another arena in which one might expect BON's may arise, and it is possible that in such vacua one could study decays with stabilized moduli in a controlled approximation. One might guess that the existence of such solutions would depend on whether the theory, in the limit that certain moduli become large, is or is not supersymmetric. Such an analysis will also force us to sharpen the distinction between supersymmetric and non-supersymmetric strings. Our work also suggests that such solutions will exist in the case of Randall-Sundrum [14, 15] type models of warped dimensions.

In light of these observations, reasonable to suggest that broad classes of non-supersymmetric vacua do not make sense. This might be the basis of a prediction of supersymmetry in string theory. However we have noted alternative interpretations of these solutions which some readers may find quite plausible. These issues seem sufficiently intriguing — and potentially important — to be worthy of further study.

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